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OSp(4|2) superconformal mechanics

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Abstract: A new superconformal mechanics with OSp(4|2) symmetry is obtained by gauging the U(1) isometry of a superfield model. It is the one-particle case of the new $\mathcal{N}$=4 super Calogero model recently proposed in arXiv:0812.4276 [hep-th]. Classical and quantum generators of the $osp(4|2)$ superalgebra are constructed on physical states. As opposed to other realizations of $\mathcal{N}$=4 superconformal algebras, all supertranslation generators are linear in the odd variables, similarly to the $\mathcal{N}$=2 case. The bosonic sector of the component action is standard one-particle (dilatonic) conformal mechanics accompanied by an SU(2)/U(1) Wess-Zumino term, which gives rise to a fuzzy sphere upon quantization. The strength of the conformal potential is quantized.

Keywords: Extended Supersymmetry, Superspaces, Integrable Equations in Physics, Black Holes in String Theory

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Contents

1 Introduction

The stable interest in conformal mechanics [1–9] and its various superconformal extensions [4–7, 9–22] is caused by two closely connected reasons. First, these models describe (super)particles moving in near-horizon (AdS) geometries of black-hole solutions of supergravities in diverse dimensions and so bear an intimate relation to the AdS/CFT correspondence. Second, they are one-particle prototypes of many-particle d=1 integrable (super)conformal systems of the Calogero type, which are the object of numerous studies. The search for new models of this kind and their implications in the areas just mentioned present interesting venues for study.

It has been proposed in [4] that the radial motion of a massive charged particle near the horizon of an extremal Reissner-Nordström (RN) black hole is described by conformal mechanics [1]. The target variable of this conformal mechanics is the AdS_2 radial coordinate as part of the $\mathrm{AdS}_2\times S^2$ background. The latter is the bosonic body of the maximally supersymmetric near-horizon extremal RN (Reissner-Nordström) solution of $\mathcal{N}$=2 D=4 supergravity [4, 23], with the full isometry supergroup $\mathrm{SU}(1,1|2)$. Based on this observation, it was suggested in [4] that the $\mathrm{SU}(1,1|2)$ $\mathcal{N}$=4 superconformal mechanics describes the full dynamics of a superparticle in the near-horizon geometry of extremal RN black holes.

SU(1,1|2) superconformal mechanics was constructed and investigated more than twenty years ago in [12] in the framework of the nonlinear realizations approach. In [13], some of the results of [12] were rediscovered and transported into the modern black-hole and AdS/CFT context. In [5], it was then argued that an n-particle generalization of the SU(1,1|2) superconformal mechanics, in the form of a superconformal Calogero model, in the large-n limit provides a microscopic description of *multiple* extremal RN black holes in the near-horizon limit. Further evidence in favor of the proposal of [4] was adduced in [9, 15], where a canonical transformation was found to link the radial motion of a (super)particle on $\mathrm{AdS}_2 \times S^2$ as bosonic background with $\mathcal{N}$=0, $\mathcal{N}$=2 [9] and $\mathcal{N}$=4 [15] superconformal mechanics.

There are good reasons to look beyond SU(1,1|2) to the most general $\mathcal{N}$=4 superconformal group in one dimension, which is the exceptional one-parameter supergroup $D(2,1;\alpha)$ [24]. It reduces to SU(1,1|2)$\otimes$SU(2) at α=0 and $\alpha=-1$. In fact, the isometry supergroup of a near-horizon M-brane solution of D=11 supergravity was determined as $D(2,1;\alpha)\times D(2,1;\alpha)$ [25], and $D(2,1;\alpha)$ is physically realized for any value of the parameter α in the near-horizon M-theory solutions [26].

The general one-dimensional sigma model with $D(2,1;\alpha)$ supersymmetry, in terms of $\mathcal{N}$=1 superfields, was applied in [7] to the non-relativistic spinning particle propagating in a curved background augmented with a magnetic field and a scalar potential. $D(2,1;\alpha)$ superconformal mechanics was also constructed in [16] in the nonlinear realizations superfield framework. Described by the $(\mathbf{3},\mathbf{4},\mathbf{1})$ off-shell $\mathcal{N}$=4 supermultiplet, this model contains in its bosonic sector three fields, which stand for the dilaton and for the coordinates of the coset $S^2 \simeq \mathrm{SU}(2)/\mathrm{U}(1)$, thus governing a particle moving on $\mathrm{AdS}_2\times S^2$. Furthermore, with the help of a special canonical transformation, a recent paper [20] established a connection between the model of [16] with $D(2,1;-1) \simeq$ SU(1,1|2)$\otimes$SU(2) invariance and a particle propagating near the horizon of extremal RN black hole with magnetic charge.

In the present paper we construct and examine a new type of $\mathcal{N}$=4 superconformal mechanics model, which is invariant under the supergroup $D(2,1;-\frac{1}{2}) \simeq \mathrm{OSp}(4|2)$. We note that OSp(4|2) is distinguished among all $\mathcal{N}$=4 supergroups $D(2,1;\alpha)$ because its coset superspace $\mathrm{OSp}(4|2)/[\mathrm{SO}(1,1)\times\mathrm{SO}(2)\times\mathrm{SU}(2)]$ is the only superextension of $\mathrm{AdS}_2\times S^2$ which admits a superconformally flat supervielbein and superconnections, as opposed to the more conventional coset superspace $\mathrm{SU}(1,1|2)/[\mathrm{SO}(1,1)\times\mathrm{SO}(2)]$ [27].

Our new OSp(4|2) mechanics arises as the n=1 case of an $\mathcal{N}$=4 supersymmetric generalization of the A_{n-1} Calogero system proposed recently in [28], and it radically differs from the model of [16]. For one, our model is defined by a *reducible* $D(2,1;-\frac{1}{2})$ representation, namely it is a coupled system of one $(\mathbf{1},\mathbf{4},\mathbf{3})$ multiplet and one $(\mathbf{4},\mathbf{4},\mathbf{0})$ multiplet, both presented by appropriate bosonic superfields. In the action, the $(\mathbf{4},\mathbf{4},\mathbf{0})$ multiplet is described by a pure superfield Wess-Zumino term, without standard kinetic term. Furthermore, our model posesses a gauged U(1) symmetry, ensured by a non-propagating gauge multiplet. After fixing this U(1) in a manifestly $\mathcal{N}$=4 supersymmetric way, the $(\mathbf{4},\mathbf{4},\mathbf{0})$ Wess-Zumino multiplet turns into a $(\mathbf{3},\mathbf{4},\mathbf{1})$ multiplet, which superconformally couples to the $(\mathbf{1},\mathbf{4},\mathbf{3})$ superfield. Alternatively, a Wess-Zumino gauge choice may be more suitable for analyzing the component structure and for its quantization.

In the next two sections we give a general description of the model, first in superfields and then in component fields. Quantization is performed in section 4. We employ the harmonic framework of [17, 29]. Thus, from the very beginning, the super worldline is extended by SU(2)/U(1) harmonics. After eliminating auxiliary fields in the component action, we obtain harmonic-like fields also in the target space. The action for these fields is only of first order in time derivatives, hence get quantized to pure spin (or "isospin") degrees of freedom. Thus, starting from a theory with worldline harmonic variables, we arrive at a sort of harmonic target superspace. The corresponding wave functions are irreducible SU(2) multispinors, in contradistinction to ordinary conformal or superconformal quantum mechanics [1, 10–13] where spin is solely due to the fermionic fields and disappears in the bosonic limit. Here instead, the bosonic quantum sector may be interpreted as a direct product of standard quantum conformal mechanics [1] with a *fuzzy sphere* [30], which appears by virtue of the S^2 Wess-Zumino term.

2 Superfield setup

A natural arena for $\mathcal{N}$=4, d=1 supersymmetric theories is the $\mathcal{N}$=4, d=1 superspace [12]

$$(t, \theta_i, \bar\theta^i)\,, \qquad \bar\theta^i = (\overline{\theta_i})\,, \quad (i = 1, 2)\,. \tag{2.1}$$

The corresponding spinor covariant derivatives have the form

$$D^i = \frac{\partial}{\partial\theta_i} + i\bar\theta^i\partial_t\,, \qquad \bar D_i = \frac{\partial}{\partial\bar\theta^i} + i\theta_i\partial_t = -\overline{(D^i)}\,.$$

The full R-symmetry (automorphism) group of (2.1) is SO$(4)_R$. One of the two SU(2) factors of the latter acts on the doublet indices i and will be denoted SU$(2)_R$. The second SU(2) mixes θ_i with their complex conjugates and is not manifest in the considered approach.

Off-shell $\mathcal{N}$=4, d=1 supermultiplets admit a concise formulation in the harmonic superspace (HSS) [16], an extension of (2.1) by the harmonic coordinates $u^\pm_i$:

$$(t, \theta^\pm, \bar\theta^\pm, u^\pm_i)\,, \qquad \theta^\pm = \theta^i u^\pm_i\,, \qquad \bar\theta^\pm = \bar\theta^i u^\pm_i\,, \qquad u^{+i}u^-_i = 1\,. \tag{2.2}$$

The commuting SU(2) spinors $u^\pm_i$ parametrize the 2-sphere $S^2 \sim \mathrm{SU}(2)_R/\mathrm{U}(1)_R$. The salient property of HSS is the presence of an important subspace in it, the harmonic analytic superspace (ASS) with half of Grassmann co-ordinates as compared to (2.1) or (2.2):

$$(\zeta, u) = (t_A, \theta^+, \bar\theta^+, u^\pm_i)\,, \qquad t_A = t - i(\theta^+\bar\theta^- + \theta^-\bar\theta^+)\,. \tag{2.3}$$

It is closed under the $\mathcal{N}$=4 supersymmetry transformations. Most of the off-shell $\mathcal{N}$=4, d=1 multiplets are represented by the analytic superfields, i.e. those "living" on (2.3).

Spinor covariant derivatives in the analytic basis of HSS, viz. $(\zeta, u, \theta^-, \bar\theta^-)$, take the form

$$D^+ = \frac{\partial}{\partial\theta^-}\,, \quad \bar D^+ = -\frac{\partial}{\partial\bar\theta^-}\,, \qquad D^- = -\frac{\partial}{\partial\theta^+} + 2i\bar\theta^-\partial_{t_A}\,, \quad \bar D^- = \frac{\partial}{\partial\bar\theta^+} + 2i\theta^-\partial_{t_A}\,. \tag{2.4}$$

In the central basis (2.2), the same derivatives are defined as the projections $D^{\pm} = D^i u_i^{\pm}$ and $\bar{D}^{\pm} = \bar{D}^i u_i^{\pm}$. Harmonic covariant derivatives in the analytic basis read

$$D^{\pm\pm} = \partial^{\pm\pm} - 2i\theta^{\pm}\bar{\theta}^{\pm}\partial_{t_A} + \theta^{\pm}\frac{\partial}{\partial\theta^{\mp}} + \bar{\theta}^{\pm}\frac{\partial}{\partial\bar{\theta}^{\mp}}\,. \tag{2.5}$$

The integration measures are defined by

$$\mu_H = dudtd^4\theta = \mu_A^{(-2)}(D^+\bar{D}^+)\,, \qquad \mu_A^{(-2)} = du d\zeta^{(-2)} = dudt_A d\theta^+ d\bar{\theta}^+ = dudt_A(D^-\bar{D}^-)\,.$$

2.1 Action

In [28], we constructed a new $\mathcal{N}$=4 supersymmetric extension of the A_{n-1} Calogero system. Distinguishing features of its Lagrangian are, first, the appearance of the U(2) spin generalization of the A_{n-1} Calogero in its bosonic sector, second, $\mathcal{N}$=4 superconformal invariance associated with the supergroup $D(2,1;-\frac{1}{2}) \simeq \mathrm{OSp}(4|2)$ (as opposed to the $\mathrm{SU}(1,1|2)$ superconformal symmetry of the standard $\mathcal{N} = 4$ superextensions [12, 13]) and, third, a nontrivial coupling to the center-of-mass coordinate. All these features are retained even in the extremal n=1 case where only the center-of-mass coordinate is present. It develops a conformal potential, so the n=1 case of the $\mathcal{N}$=4 Calogero model of [28] amounts to a non-trivial model of $\mathcal{N}$=4 superconformal mechanics (as distinct from the new $\mathcal{N}$=1, 2 models also obtained in [28] by the same method; in them, the n=1 case yields a free system). Below we describe the superfield action of this model.

It involves superfields corresponding to three off-shell $\mathcal{N}$=4 supermultiplets: (**i**) the "radial" multiplet (**1,4,3**); (**ii**) the Wess-Zumino ("isospin") multiplet (**4,4,0**); and (**iii**) the gauge ("topological") multiplet (**0,0,0**). The action has the form

$$S = S_{\mathcal{X}} + S_{FI} + S_{WZ}\,. \tag{2.6}$$

First term in (2.6) is the standard free action of (**1,4,3**) multiplet

$$S_{\mathcal{X}} = -\frac{1}{2}\int \mu_H\, \mathcal{X}^2\,, \tag{2.7}$$

where the even real superfield $\mathcal{X}$ is subjected to the constraints

$$D^{++}\,\mathcal{X} = 0\,, \tag{2.8}$$

$$D^+D^-\,\mathcal{X} = 0\,, \qquad \bar{D}^+\bar{D}^-\,\mathcal{X} = 0\,, \qquad (D^+\bar{D}^- + \bar{D}^+D^-)\,\mathcal{X} = 0\,. \tag{2.9}$$

The set of conditions (2.8) and (2.9) is equivalent to the standard constraints $D^iD_i\,\mathcal{X} = 0$, $\bar{D}_i\bar{D}^i\,\mathcal{X} = 0$, $[D^i, \bar{D}_i]\,\mathcal{X} = 0$ in the central basis (2.2).

Second term in (2.6) is Fayet-Iliopoulos (FI) term

$$S_{FI} = \frac{i}{2}\,c\int \mu_A^{(-2)}\,V^{++} \tag{2.10}$$

for the gauge supermultiplet. The even analytic gauge superfield $V^{++}(\zeta, u)$, $D^+V^{++} = 0$, $\bar{D}^+V^{++} = 0$, is subjected to the gauge transformations

$$V^{++\prime} = V^{++} - D^{++}\lambda, \quad \lambda = \lambda(\zeta, u)\,, \tag{2.11}$$

JHEP08(2009)081

which are capable to gauge away, *locally*, all the components from V^{++}. However, the latter contains a component which cannot be gauged away *globally*. This is the reason why this $d=1$ supermultiplet was called “topological” in [31].

Last term in (2.6) is Wess-Zumino (WZ) term

$$S_{WZ} = \frac{1}{2}\int \mu_A^{(-2)}\, \mathcal{V}\, \bar{\mathcal{Z}}^+\, \mathcal{Z}^+\,. \tag{2.12}$$

Here, the complex analytic superfield $\mathcal{Z}^+, \bar{\mathcal{Z}}^+$ $(D^+\mathcal{Z}^+ = \bar{D}^+\mathcal{Z}^+ = 0)$, is subjected to the harmonic constraints

$$\mathcal{D}^{++}\,\mathcal{Z}^+ \equiv (D^{++} + i\,V^{++})\,\mathcal{Z}^+ = 0\,, \qquad \mathcal{D}^{++}\,\bar{\mathcal{Z}}^+ \equiv (D^{++} - i\,V^{++})\,\bar{\mathcal{Z}}^+ = 0 \tag{2.13}$$

and describes a gauge-covariantized version of the $\mathcal{N}$=4 multiplet (**4,4,0**). The relevant gauge transformations are

$$\mathcal{Z}^{+\prime} = e^{i\lambda}\mathcal{Z}^+, \qquad \bar{\mathcal{Z}}^{+\prime} = e^{-i\lambda}\bar{\mathcal{Z}}^+\,. \tag{2.14}$$

The superfield $\mathcal{V}(\zeta,u)$ in (2.12) is a real analytic gauge superfield $(D^+\,\mathcal{V} = \bar{D}^+\,\mathcal{V} = 0)$, which is a prepotential solving the constraints (2.8) for $\mathcal{X}$. It is related to the superfield $\mathcal{X}$ in the central basis by the harmonic integral transform [32]

$$\mathcal{X}(t,\theta_i,\bar{\theta}^i) = \int du\, \mathcal{V}\left(t_A,\theta^+,\bar{\theta}^+,u^\pm\right)\Big|_{\theta^\pm=\theta^i u_i^\pm,\ \bar{\theta}^\pm=\bar{\theta}^i u_i^\pm}\,. \tag{2.15}$$

The unconstrained analytic prepotential $\mathcal{V}$ has its own pregauge freedom

$$\delta\mathcal{V} = D^{++}\lambda^{--}\,, \quad \lambda^{--} = \lambda^{--}(\zeta,u)\,, \tag{2.16}$$

which can be exploited to show that $\mathcal{V}$ describes just the multiplet $(\mathbf{1},\mathbf{4},\mathbf{3})$ (after choosing the appropriate Wess-Zumino gauge) [32]. The coupling to the multiplet $(\mathbf{1},\mathbf{4},\mathbf{3})$ in (2.12) is introduced for ensuring superconformal invariance. As we shall see, upon passing to components, it gives rise to non-trivial interactions for the physical fields. The invariance of (2.12) under (2.16) is ensured by the constraints (2.13).

2.2 Superconformal invariance

Besides the gauge U(1) symmetry (2.11), (2.14) and pregauge symmetry (2.16), the action (2.6) is invariant under the rigid $\mathcal{N}$=4 superconformal symmetry $D(2,1;\alpha)$ with $\alpha = -1/2$. All superconformal transformations are contained in the closure of the supertranslations and superconformal boosts.

Invariance of the action (2.6) under the supertranslations $(\bar{\varepsilon}^i = \overline{(\varepsilon_i)})$

$$\delta t = i(\theta_k\bar{\varepsilon}^k - \varepsilon_k\bar{\theta}^k), \qquad \delta\theta_k = \varepsilon_k, \qquad \delta\bar{\theta}^k = \bar{\varepsilon}^k$$

is automatic because we use the $\mathcal{N}$=4 superfield approach.

The coordinate realization of the superconformal boosts of $D(2,1;\alpha)$ [17, 31] specialized to the case of $\alpha=-1/2$ is as follows ($\bar\eta^i=\overline{(\eta_i)}$):

$$\delta' t = -\Lambda_0 t\,, \qquad \delta'\theta_i = \eta_i t - \Lambda_0\theta_i\,, \qquad \delta'\bar\theta^i = \bar\eta^i t - \Lambda_0\bar\theta^i\,, \tag{2.17}$$

$$\delta' t_A = -2\Lambda t_A\,, \quad \delta'\theta^+ = \eta^+ t_A + i\eta^-\theta^+\bar\theta^+, \quad \delta'\bar\theta^+ = \bar\eta^+ t_A + i\bar\eta^-\theta^+\bar\theta^+, \quad \delta' u_i^+ = \Lambda^{++}u_i^-\,, \tag{2.18}$$

$$\delta'(dtd^4\theta) = 2(dtd^4\theta)\Lambda_0\,, \quad \delta'\mu_H = \mu_H\,(2\Lambda+\Lambda_0)\,, \quad \delta'\mu_A^{(-2)} = 0\,, \quad \delta' D^{++} = -\Lambda^{++}D^0, \tag{2.19}$$

where

$$\Lambda = \tilde\Lambda = i(\eta^-\bar\theta^+ - \bar\eta^-\theta^+)\,, \quad \Lambda^{++} = D^{++}\Lambda = i(\eta^+\bar\theta^+ - \bar\eta^+\theta^+)\,, \quad D^{++}\Lambda^{++} = 0\,, \tag{2.20}$$

$$\Lambda_0 = 2\Lambda - D^{--}\Lambda^{++} = i(\eta_k\bar\theta^k + \bar\eta^k\theta_k)\,, \qquad D^{++}\Lambda_0 = 0\,. \tag{2.21}$$

Taking the field transformations in the form (here we use the "passive" interpretation of them)

$$\delta'\mathcal{X} = -\Lambda_0\,\mathcal{X}\,, \qquad \delta'\mathcal{V} = -2\Lambda\,\mathcal{V}\,, \qquad \delta'\mathcal{Z}^+ = \Lambda\,\mathcal{Z}^+\,, \qquad \delta' V^{++} = 0\,, \tag{2.22}$$

it is easy to check the invariance of the action (2.6). Note that the constraints (2.8), (2.9) and (2.13) as well as the actions (2.10) and (2.12), are invariant with respect to the $D(2,1;\alpha)$ transformations with an arbitrary α. It is important, that the action (2.12) is superconformally invariant just due to the presence of the analytic prepotential $\mathcal{V}$. The free action (2.7) is invariant only under the supergroup $D(2,1;\alpha=-1/2)\sim \mathrm{OSp}(4|2)$ which is thus the superconformal symmetry of the full action (2.6).

2.3 Supersymmetric gauge

In the next sections we will analyse the component structure of the model by choosing the Wess-Zumino (WZ) gauge for the superfield V^{++}. However, in order to clarify the off-shell superfield content of our model, it is instructive to fix the underlying U(1) gauge freedom by choosing a gauge which preserve manifest $\mathcal{N}$=4 supersymmetry. A gauge suitable for our purpose was used in [31].

To make contact with the consideration in [31], let us combine the superfields $\mathcal{Z}^+$ and $\bar{\mathcal{Z}}^+$ into a doublet of some extra ("Pauli-Gürsey") $\mathrm{SU}(2)_{PG}$ group as

$$q^{+a} := (\bar{\mathcal{Z}}^+, \mathcal{Z}^+)\,,\; a=1,2 \tag{2.23}$$

and rewrite the transformation law (2.14) and the constraints (2.13) as

$$\delta q^{+a} = \lambda c^a_{\ b} q^{+b}\,, \quad D^{++}q^{+a} + V^{++}c^a_{\ b}q^{+b} = 0\,. \tag{2.24}$$

Here, the traceless constant tensor $c^a_{\ b}$ breaks $\mathrm{SU}(2)_{PG}$ down to U(1) which is just the symmetry to be gauged. Choosing the frame where the only non-zero entries of $c^a_{\ b}$ are $c^1_{\ 1} = -c^2_{\ 2} = -i$, we recover the transformation law (2.14) and the constraints (2.13). It is easy to see that

$$\bar{\mathcal{Z}}^+\mathcal{Z}^+ = -\frac{i}{2}\,q^{+a}\,c_{ab}\,q^{+b}\,. \tag{2.25}$$

In [31] (following [29]) an invertible equivalence redefinition of $q^{+a} \Rightarrow (\omega, l^{++})$ has been exploited, such that the U(1) gauge transformation in (2.24) is realized as

$$\delta\omega = -2\lambda\,, \quad \delta l^{++} = 0 \tag{2.26}$$

(the precise form of this equivalence transformation is given in eq. (4.26) in [31]; it is a superfield analog of the standard polar decomposition of a vector). Then one can fully fix the U(1) gauge freedom by imposing the manifestly $\mathcal{N}$=4 supersymmetric gauge

$$\omega = 0\,. \tag{2.27}$$

In this gauge, the harmonic constraint in (2.24) implies

$$\text{(a)} \qquad q^{+a}\, c_{ab}\, q^{+b} = 4(c^{++} + l^{++})\,, \qquad \text{(b)}\ V^{++} = \frac{l^{++}}{(1+\sqrt{1+c^{--}l^{++}})\sqrt{1+c^{--}l^{++}}}\,,$$
$$\text{(c)}\ D^{++}(c^{++} + l^{++}) = D^{++}l^{++} = 0\,, \tag{2.28}$$

where $c^{\pm\pm} = c^{(ab)}u^{\pm}_a u^{\pm}_b$. After substituting the expressions (2.28a) and (2.28b) into (2.12) and (2.10), the total superfield action (2.6) takes the form:

$$S = -\frac{1}{2}\int \mu_H\, \mathcal{X}^2 - i\int \mu^{(-2)}_A \left[\mathcal{V}\,(c^{++} + l^{++}) - \frac{c}{2}\,\frac{l^{++}}{(1+\sqrt{1+c^{--}l^{++}})\sqrt{1+c^{--}l^{++}}}\right]. \tag{2.29}$$

The superfield l^{++} with the constraint (2.28c) accommodates an off-shell $\mathcal{N}$=4 multiplet $(\mathbf{3},\mathbf{4},\mathbf{1})$ [17]. So, the action (2.29) describes a system of two interacting off-shell $\mathcal{N}$=4, d=1 multiplets: $(\mathbf{1},\mathbf{4},\mathbf{3})$ represented by the superfield $\mathcal{X}$ and $(\mathbf{3},\mathbf{4},\mathbf{1})$ represented by the analytic superfield l^{++}. This is the off-shell content of our OSp(4|2) model. As distinct from the superconformal mechanics based on a single $(\mathbf{3},\mathbf{4},\mathbf{1})$ multiplet the action of which is a sum of the sigma-model type term and WZ term of l^{++} [16, 17], the action (2.29) involves only conformal superfield WZ term of this multiplet (the last term in the square brackets). The interaction with the multiplet $(\mathbf{1},\mathbf{4},\mathbf{3})$ is accomplished through a superconformal bilinear coupling of both multiplets (the first term in the square brackets).[1] Notice that, due to the absence of the kinetic term for l^{++} in (2.29), the on-shell content of the model appears to be drastically different from the off-shell one: the eventual component action contains only three bosonic fields and four fermionic fields, which are joined into some new on-shell $(\mathbf{3},\mathbf{4},\mathbf{1})$ multiplet (see the next section).

3 Component actions

3.1 Action for (1,4,3) supermultiplet

The solution of the constraint (2.8), (2.9) is as follows (in the analytic basis):

$$\mathcal{X} = x + \theta^-\psi^+ + \bar\theta^-\bar\psi^+ - \theta^+\psi^- - \bar\theta^+\bar\psi^- + \theta^-\bar\theta^-N^{++} + \theta^+\bar\theta^+N^{--} + (\theta^-\bar\theta^+ + \theta^+\bar\theta^-)N$$
$$+\,\theta^-\theta^+\bar\theta^-\Omega^+ + \bar\theta^-\bar\theta^+\theta^-\bar\Omega^+ + \theta^-\bar\theta^-\theta^+\bar\theta^+D\,, \tag{3.1}$$

[1] The existence of such a coupling and its potential implications in the models of superconformal $\mathcal{N}$=4 mechanics were noted for the first time in [32].

where

$$N^{\pm\pm} = N^{ik}u_i^{\pm}u_k^{\pm}\,, \qquad N = i\dot{x} - N^{ik}u_i^{+}u_k^{-}\,, \qquad D = 2\ddot{x} + 2i\dot{N}^{ik}u_i^{+}u_k^{-}\,, \tag{3.2}$$

$$\psi^{\pm} = \psi^{i}u_i^{\pm}\,, \qquad \bar{\psi}^{\pm} = \bar{\psi}^{i}u_i^{\pm}\,, \qquad \Omega^{+} = 2i\dot{\psi}^{+}\,, \qquad \bar{\Omega}^{+} = -2i\dot{\bar{\psi}}^{+} \tag{3.3}$$

and $x(t_A)$, $N^{ik} = N^{(ik)}(t_A)$, $\psi^i(t_A)$, $\bar{\psi}_i(t_A) = (\overline{\psi^i})$ are d=1 fields.

Inserting (3.1) in (2.7) and integrating there over the θ- and harmonic variables,[2] we obtain

$$S_{\mathcal{X}} = \int dt \left[\dot{x}\dot{x} - i\left(\bar{\psi}_k\dot{\psi}^k - \dot{\bar{\psi}}_k\psi^k\right) - \frac{1}{2}N^{ik}N_{ik}\right]. \tag{3.4}$$

In the central basis the θ expansion (3.1) takes the form:

$$\mathcal{X}(t,\theta_i,\bar{\theta}^i) = x + \theta_i\psi^i + \bar{\psi}_i\bar{\theta}^i + \theta^i\bar{\theta}^k N_{ik} + \frac{i}{2}(\theta)^2\dot{\bar{\psi}}_i\bar{\theta}^i + \frac{i}{2}(\bar{\theta})^2\theta_i\dot{\bar{\psi}}^i + \frac{1}{4}(\theta)^2(\bar{\theta})^2\ddot{x} \tag{3.5}$$

where $(\theta)^2 \equiv \theta_i\theta^i = -2\theta^+\theta^-$, $(\bar{\theta})^2 \equiv \bar{\theta}^i\bar{\theta}_i = 2\bar{\theta}^+\bar{\theta}^-$. Then, from (2.15) we can identify the fields appearing in the WZ gauge for $\mathcal{V}$ with the fields in (3.5)

$$\mathcal{V}(t_A,\theta^+,\bar{\theta}^+,u^{\pm}) = x(t_A) - 2\,\theta^+\psi^i(t_A)u_i^- - 2\,\bar{\theta}^+\bar{\psi}^i(t_A)u_i^- + 3\,\theta^+\bar{\theta}^+N^{ik}(t_A)u_i^-u_k^-\,. \tag{3.6}$$

This expansion will be used to express the action (2.12) in terms of the component fields.

3.2 FI and WZ actions

Using the U(1) gauge freedom (2.11), (2.14) we can choose WZ gauge

$$V^{++} = -2i\,\theta^+\bar{\theta}^+A(t_A)\,. \tag{3.7}$$

Then

$$S_{FI} = c\int dt\,A\,. \tag{3.8}$$

The solution of the constraint (2.13) in WZ gauge (3.7) is

$$\mathcal{Z}^+ = z^iu_i^+ + \theta^+\varphi + \bar{\theta}^+\phi + 2i\,\theta^+\bar{\theta}^+\nabla_{t_A}z^iu_i^-\,, \qquad \bar{\mathcal{Z}}^+ = \bar{z}_iu^{+i} + \theta^+\bar{\phi} - \bar{\theta}^+\bar{\varphi} + 2i\,\theta^+\bar{\theta}^+\nabla_{t_A}\bar{z}_iu^{-i} \tag{3.9}$$

where

$$\nabla z^k = \dot{z}^k + iA\,z^k\,, \qquad \nabla\bar{z}_k = \dot{\bar{z}}_k - iA\,\bar{z}_k\,. \tag{3.10}$$

In (3.9), $z^i(t_A)$ and $\varphi(t_A)$, $\phi(t_A)$ are d=1 fields, bosonic and fermionic, respectively. The fields z^i form a complex doublet of the R-symmetry SU(2) group, while the fermionic fields are singlets of the latter. Another ("mirror") R-symmetry SU(2) is not manifest in the present approach: the bosonic fields are its singlets, while the fermionic fields form a doublet with respect to it.

[2] Here the harmonics integrals $\int du\,u^{+i}u_k^- = \frac{1}{2}\,\delta_k^i$, $\int du\,u^{+(i_1}u^{+i_2)}u^-_{(k_1}u^-_{k_2)} = -2\int du\,u^{+(i_1}u^{-i_2)}u^+_{(k_1}u^-_{k_2)}$ $= \frac{1}{3}\,\delta^{(i_1}_{(k_1}\delta^{i_2)}_{k_2)}$ are used.

Inserting expressions (3.9) and (3.6) in the action (2.12) and integrating over θ s and harmonics, we obtain a component form of the WZ action

$$S_{WZ} = \frac{i}{2}\int dt\,\left(\bar z_k \nabla z^k - \nabla \bar z_k\, z^k\right) x - \frac{1}{2}\int dt\, N^{ik}\bar z_i z_k \tag{3.11}$$
$$+\frac{1}{2}\int dt\left[\psi^k\left(\bar\varphi\, z_k + \bar z_k\phi\right) + \bar\psi^k\left(\bar\phi\, z_k - \bar z_k\varphi\right) - x\left(\bar\phi\,\phi + \bar\varphi\,\varphi\right)\right].$$

The fermionic fields ϕ, φ are auxiliary. The action is invariant under the residual local U(1) transformations

$$A' = A - \dot\lambda_0\,, \quad z^{i\prime} = e^{i\lambda_0} z^i\,,\ \bar z_i{}' = e^{-i\lambda_0}\bar z_i \tag{3.12}$$

(and similar phase transformations of the fermionic fields).

The total component action is a sum of (3.4), (3.8) and (3.11). Eliminating the auxiliary fields N^{ik}, ϕ, $\bar\phi$, φ, $\bar\varphi$, from this sum by their algebraic equations of motion,

$$N_{ik} = -\frac{1}{2}\, z_{(i}\bar z_{k)}\,, \tag{3.13}$$
$$\phi = -\frac{\bar\psi^k z_k}{x}\,, \quad \bar\phi = \frac{\psi^k \bar z_k}{x}\,, \quad \varphi = -\frac{\psi^k z_k}{x}\,, \quad \bar\varphi = -\frac{\bar\psi^k \bar z_k}{x}\,, \tag{3.14}$$

and making the redefinition

$$z'^i = x^{1/2}\, z^i\,, \tag{3.15}$$

we obtain that the action (2.6) in WZ gauge takes the following on-shell form (we omitted the primes on z)

$$S = S_b + S_f\,, \tag{3.16}$$
$$S_b = \int dt\left[\dot x\dot x + \frac{i}{2}\left(\bar z_k \dot z^k - \dot{\bar z}_k z^k\right) - \frac{(\bar z_k z^k)^2}{16x^2} - A\left(\bar z_k z^k - c\right)\right], \tag{3.17}$$
$$S_f = -i\int dt\,\left(\bar\psi_k\dot\psi^k - \dot{\bar\psi}_k\psi^k\right) - \int dt\,\frac{\psi^i\bar\psi^k z_{(i}\bar z_{k)}}{x^2}\,. \tag{3.18}$$

It is still invariant under the gauge transformations (3.12). The d=1 connection $A(t)$ in (3.17) is the Lagrange multiplier for the constraint

$$\bar z_k z^k = c\,. \tag{3.19}$$

After varying with respect to A, the action (3.16) is gauge invariant only with taking into account this algebraic constraint which is gauge invariant by itself. It is convenient to fully fix the residual gauge freedom by choosing the phases of z^1 and z^2 opposite to each other. In this gauge, the constraint (3.19) is solved by

$$z^1 = \kappa\cos\frac{\gamma}{2}\, e^{i\alpha/2}\,, \qquad z^2 = \kappa\sin\frac{\gamma}{2}\, e^{-i\alpha/2}\,, \qquad \kappa^2 = c\,. \tag{3.20}$$

In terms of the newly introduced fields the action (3.16) takes the form

$$S = S_b + S_f\,, \tag{3.21}$$

$$S_b = \int dt \left[\dot{x}\dot{x} - \frac{c^2}{16\,x^2} - \frac{c}{2}\cos\gamma\,\dot{\alpha}\right], \tag{3.22}$$

$$\begin{aligned} S_f =& -i\int dt\,\left(\bar{\psi}_k\dot{\psi}^k - \dot{\bar{\psi}}_k\psi^k\right) \\ &+ \frac{c}{2}\int dt\,\frac{\cos\gamma\left(\psi^1\bar{\psi}_1 + \psi^2\bar{\psi}_2\right) - \sin\gamma\left(e^{i\alpha}\psi^2\bar{\psi}_1 + e^{-i\alpha}\psi^1\bar{\psi}_2\right)}{x^2}\,. \end{aligned} \tag{3.23}$$

Unconstrained fields in the action (3.21), three bosons x, γ, α and four fermions ψ^k, $\bar{\psi}_k$, constitute some on-shell (**3,4,1**) supermultiplet. As opposed to the (**3,4,1**) supermultiplet considered in [12, 16, 18] the action (3.22) contains "true" kinetic term only for one bosonic component which also possesses the conformal potential, whereas two other fields parametrizing the coset $\mathrm{SU}(2)_R/\mathrm{U}(1)_R$ are described by a WZ term and so become a sort of isospin degrees of freedom (target SU(2) harmonics) upon quantization. The realization of OSp(4|2) superconformal transformations on these fields will be given in the next section.

It should be stressed that the considered model realizes a new mechanism of generating conformal potential $\sim 1/x^2$ for the field $x(t)$. Before eliminating auxiliary fields, the component action contains no explicit term of this kind. It arises as a result of varying with respect to the Lagrange multiplier $A(t)$ and making use of the arising constraint (3.19). As we shall see soon, in quantum theory this new mechanism entails a quantization of the constant c. In the SU(1,1|2) superconformal quantum mechanics, the strength of the conformal potential appears in the $su(1,1|2)$ algebra as a constant central charge [12, 14, 19]. In our model such an option does not exist since the superalgebra $osp(4|2)$ does not alow a central extension.

Notice that an equivalent component action can be obtained starting from the superfield action (2.29) which corresponds to another choice of the gauge with respect to U(1) transformations. As distinct from the WZ gauge used in this section, the gauge corresponding to (2.29) preserves the manifest $\mathcal{N}$=4 supersymmetry and does not exhibit any residual gauge freedom. The component bosonic sector of (2.29) involves one physical $x = \mathcal{X}|_{\theta=0}$ and three bosonic fields $y^{(ik)}$ from the $(\mathbf{3},\mathbf{4},\mathbf{1})$ superfield l^{++}. They form a 3-vector with respect to $\mathrm{SU}(2)_R$ ($l^{++} + c^{++} = y^{(ik)}u^+_i u^+ + \theta$-dependent terms). By an algebraic constraint, with the auxiliary field of $(\mathbf{3},\mathbf{4},\mathbf{1})$ as a Lagrange multiplier, the fields $y^{(ik)}$ are confined to parametrize a sphere S^2. This constraint plays a role analogous to (3.19). The gauge-invariant fields $y^{(ik)}$ are related to the doublet fields $z^i, \bar{z}_k$ via the well-known first Hopf map (see also section below). The relation (2.28a) is in fact a superfield version of this map. Thus, one again ends up with 3 bosonic fields and 4 fermionic fields forming an irreducible on-shell multiplet.

It is also worth noting that this reduction of two independent off-shell $\mathcal{N}$=4 multiplets (**3,4,1**) and (**1,4,3**) to a smaller on-shell $\mathcal{N}$=4 multiplet somewhat resembles the procedure of ref. [33] in which some irreducible $\mathcal{N}$=4 multiplets with four physical fermions are generated from pairs of other multiplets of this type by identifying fermionic fields in the multiplets forming the pair. In our case such identification arises as one of the algebraic equa-

JHEP08(2009)081

tions of motion, eq. (3.14). In this connection, it would be interesting to inquire whether the component action (3.21) can be independently re-derived from an alternative (dual) superfield action corresponding to some nonlinear version of the off-shell multiplet (**3,4,1**).

3.3 $\mathcal{N}$=4 superconformal symmetry in WZ gauge

The transformations and their generators look most transparent in terms of the SU(2) doublet quantities z^k and $\bar{z}_k$.

To determine the superconformal transformations of component fields, we should know the appropriate compensating gauge transformations needed to preserve the WZ gauge (3.7). For supertranslations and superconformal boosts the parameter of the compensating gauge transformations is as follows

$$\lambda = 2i\left[(\theta^+\bar{\varepsilon}^- - \bar{\theta}^+\varepsilon^-) + t_A\,(\theta^+\bar{\eta}^- - \bar{\theta}^+\eta^-)\right]A \tag{3.24}$$

where

$$\varepsilon^- := \varepsilon^i u_i^-\,, \quad \eta^- := \eta^i u_i^-\,. \tag{3.25}$$

Taking this into account, we obtain the relevant infinitesimal OSp(4|2) transformations:

$$\delta x = -\omega_i\psi^i + \bar{\omega}^i\bar{\psi}_i\,, \tag{3.26}$$

$$\delta\psi^i = \frac{\bar{\omega}_k z^{(i}\bar{z}^{k)}}{2x} - i\bar{\omega}^i\dot{x} + i\bar{\eta}^i x\,, \tag{3.27}$$

$$\delta z^i = \frac{\omega^{(i}\psi^{k)} + \bar{\omega}^{(i}\bar{\psi}^{k)}}{x}\,z_k\,, \tag{3.28}$$

$$\delta A = 0\,, \tag{3.29}$$

where $\omega^i = \varepsilon^i + t\,\eta^i$.

Now, using the Nöther procedure, we can directly find the classical generators of the supertranslations

$$Q^i = p\,\psi^i - i\frac{z^{(i}\bar{z}^{k)}\psi_k}{x}\,, \qquad \bar{Q}_i = p\,\bar{\psi}_i + i\frac{z_{(i}\bar{z}_{k)}\bar{\psi}^k}{x} \tag{3.30}$$

where $p \equiv 2\dot{x}$, as well as of the superconformal boosts:

$$S^i = -2\,x\psi^i + t\,Q^i, \qquad \bar{S}_i = -2\,x\bar{\psi}_i + t\,\bar{Q}_i\,. \tag{3.31}$$

The remaining (even) generators of the supergroup OSp(4|2) can be found by evaluating anticommutators of the above odd generators among themselves.

As follows from the action (3.16), the SU(2) spinor variables are canonically self-conjugate due to the presence of second-class constraints for their momenta. As a result, non-vanishing canonical Dirac brackets (at equal times) have the following form

$$[x,p]_D = 1, \qquad [z^i,\bar{z}_j]_D = -i\delta^i_j, \qquad \{\psi^{ii'},\psi^{kk'}\}_D = \frac{i}{2}\,\epsilon^{ik}\epsilon^{i'k'} \tag{3.32}$$

where we introduced the notations

$$\psi^{ii'} = (\psi^{i1'},\psi^{i2'}) = (\psi^i,\bar{\psi}^i), \qquad (\overline{\psi^{ii'}}) = \psi_{ii'} = \epsilon_{ik}\epsilon_{i'k'}\psi^{kk'}\,, \quad (\epsilon_{12} = \epsilon^{21} = 1). \tag{3.33}$$

JHEP08(2009)081

Using Dirac brackets (3.32), we arrive at the following closed superalgebra:

$$\{Q^{ai'i}, Q^{bk'k}\}_D = 2i\left(\epsilon^{ik}\epsilon^{i'k'}T^{ab} + \alpha\epsilon^{ab}\epsilon^{i'k'}J^{ik} - (1+\alpha)\epsilon^{ab}\epsilon^{ik}I^{i'k'}\right), \tag{3.34}$$

$$\left[T^{ab}, T^{cd}\right]_D = -\epsilon^{ac}T^{bd} - \epsilon^{bd}T^{ac}, \quad [J^{ij}, J^{kl}]_D = -\epsilon^{ik}J^{jl} - \epsilon^{jl}J^{ik},$$

$$\left[I^{i'j'}, I^{k'l'}\right]_D = -\epsilon^{ik}I^{j'l'} - \epsilon^{j'l'}I^{i'k'}, \tag{3.35}$$

$$\left[T^{ab}, Q^{ci'i}\right]_D = \epsilon^{c(a}Q^{b)i'i}, \quad [J^{ij}, Q^{ai'k}]_D = \epsilon^{k(i}Q^{ai'j)}, \quad [J^{i'j'}, Q^{ak'i}]_D = \epsilon^{k'(i'}Q^{aj')i} \tag{3.36}$$

where $\alpha = -\frac{1}{2}$. In (3.34)–(3.36) we use the notation

$$Q^{21'i} = -Q^i, \qquad Q^{22'i} = -\bar{Q}^i, \qquad Q^{11'i} = S^i, \qquad Q^{12'i} = \bar{S}^i, \tag{3.37}$$

$$T^{22} = H, \qquad T^{11} = K, \qquad T^{12} = -D. \tag{3.38}$$

The explicit expressions for the generators are

$$H = \frac{1}{4}p^2 + \frac{(\bar{z}_k z^k)^2}{16\,x^2} + \frac{z^i\bar{z}^j\,\psi_i{}^{k'}\psi_{jk'}}{2\,x^2}, \tag{3.39}$$

$$K = x^2 - t\,xp + t^2\,H, \tag{3.40}$$

$$D = -\frac{1}{2}\,xp + t\,H, \tag{3.41}$$

$$J^{ij} = i\left[z^{(i}\bar{z}^{j)} + \psi^{ik'}\psi^j{}_{k'}\right], \tag{3.42}$$

$$I^{i'j'} = i\psi^{ki'}\psi_k{}^{j'}. \tag{3.43}$$

The relations (3.34)–(3.36) provide the standard form of the superalgebra $D(2,1;-\frac{1}{2}) \simeq \mathrm{OSp}(4|2)$ (see, e.g., [16, 24, 27]). Bosonic generators $T^{ab} = T^{ba}$, $J^{ik} = J^{ki}$, $I^{i'k'} = I^{k'i'}$ form mutually commuting $su(1,1)$, $su(2)$ and $su'(2)$ algebras, respectively.

The expression (3.39) is precisely the canonical Hamiltonian obtained from the action (3.16). Owing to the A-term in (3.16), there is also the first-class constraint

$$D^0 \equiv \bar{z}_k z^k - c \approx 0, \tag{3.44}$$

which should be imposed on wave functions in quantum case.

In the next section we shall construct a quantum realization of $D(2,1;-\frac{1}{2})$ superalgebra given above.

4 OSp(4|2) quantum mechanics

4.1 Bosonic limit and fuzzy sphere

In order to understand the specific features of our model better, we begin by quantizing it in the bosonic limit, with all fermionic fields discarded. It reveals an interesting deviation from the standard conformal quantum mechanics of de Alfaro, Fubini and Furlan [1]: besides the standard dilatonic variable $x(t)$ with the conformal potential, it also contains a fuzzy sphere [30, 34, 35] represented by the SU(2) spinor variables $z^i(t), \bar{z}^i(t)$. As a result, the

JHEP08(2009)081

relevant wave functions are non-trivial SU(2) multiplets, as opposed to the singlet wave function of the standard conformal mechanics. The strength of the conformal potential proves to coincide with the eigenvalue of the SU(2) Casimir operator (i.e. "spin") and so is quantized.

The pure bosonic model is described by the action (3.17). The corresponding canonical Hamiltonian reads

$$H_0 = \frac{1}{4}\left[p^2 + \frac{(\bar{z}_k z^k)^2}{4x^2}\right] + A\left(\bar{z}_k z^k - c\right). \tag{4.1}$$

Here $p = 2\dot{x}$ is the canonical momentum for the coordinate x. Canonical momentum for the field A is vanishing, $p_A = 0$. This constraint and the fact that the field A appears in the action (3.17) linearly, suggest to treat A as the Lagrange multiplier for the constraint

$$D^0 - c \equiv \bar{z}_k z^k - c \approx 0\,. \tag{4.2}$$

Expressions for the canonical momenta p_i and $\bar{p}^i$ for the z-variables, $[z^i, p_j]_P = \delta^i_j$, $[\bar{z}_i, p^j]_P = \delta^j_i$, are the second-class constraints

$$G_k \equiv p_k - \frac{i}{2}\,\bar{z}_k \approx 0\,, \qquad \bar{G}^k \equiv \bar{p}^k + \frac{i}{2}\,z^k \approx 0\,, \qquad [G_k, \bar{G}^l]_P = -i\delta^l_k. \tag{4.3}$$

Using Dirac brackets for them

$$[A,B]_D = [A,B]_P + i[A,G_k]_P[\bar{G}^k,B]_P - i[A,\bar{G}^k]_P[G_k,B]_P$$

we eliminate the spinor momenta p_i and $\bar{p}^i$. Dirac brackets for the residual variables are

$$[x,p]_D = 1, \qquad [z^i, \bar{z}_j]_D = -i\delta^i_j. \tag{4.4}$$

To finish with the classical description, we point out that the spinor variables describe a two-sphere. Namely, using the first Hopf map we introduce three U(1) gauge invariant variables

$$y_a = \frac{1}{2}\,\bar{z}_i(\sigma_a)^i{}_j z^j \tag{4.5}$$

where σ_a, $a = 1,2,3$ are Pauli matrices. The constraint (4.2) suggests that these variables parameterize a two-sphere with the radius $c/2$:

$$y_a y_a = (z^k \bar{z}_k)^2/4 \approx c^2/4\,. \tag{4.6}$$

The group of motion of this 2-sphere is of course the R-symmetry SU(2) group acting on the doublet indices i,k and triplet indices a. In terms of the new variables (4.5) the Hamiltonian (4.1), up to terms vanishing on the constraints, takes the form

$$H = \frac{1}{4}\left[p^2 + \frac{y_a y_a}{x^2}\right]. \tag{4.7}$$

It is worth pointing out that (4.5) is none other than the WZ gauge counterpart of the superfield Hopf map (2.28a).

At the quantum level, the algebra of the canonical operators obtained from the algebra of Dirac brackets is (quantum operators are denoted by the appropriate capital letters),

$$[X, P] = i\,, \qquad [Z^i, \bar{Z}_j] = \delta^i_j\,. \tag{4.8}$$

Then it is easy to check that the quantum counterparts of the variables (4.5)

$$Y_a = \frac{1}{2}\,\bar{Z}_i(\sigma_a)^i{}_j Z^j \tag{4.9}$$

form the SU(2) algebra

$$[Y_a, Y_b] = i\,\epsilon_{abc} Y_c\,. \tag{4.10}$$

Notice that no ordering ambiguity is present in the definition (4.9).

Moreover, the direct calculation yields

$$Y_a Y_a = \frac{1}{2}\,\bar{Z}_k Z^k \left(\frac{1}{2}\,\bar{Z}_k Z^k + 1\right) \tag{4.11}$$

and, due to the constraints (for definiteness, we adopt $\bar{Z}_k Z^k$-ordering in it), one gets

$$Y_a Y_a = \frac{c}{2}\left(\frac{c}{2} + 1\right)\,. \tag{4.12}$$

But the relations (4.10) and (4.12) are the definition of the *fuzzy sphere* coordinates [30]. Thus the angular variables, described, at the classical level, by spinor variables z^i or vector variables y_a, after quantization acquire a nice interpretation of the fuzzy sphere coordinates. Comparing the expressions (4.11) and (4.12), we observe that upon quantization the radius of the sphere changes as $\frac{c^2}{4} \to \frac{c}{2}\left(\frac{c}{2}+1\right)$.

As suggested by the relation (4.10), the fuzzy sphere coordinates Y_a are the generators of $su(2)_R$ algebra and the relation (4.12) fixes the value of its Casimir operator, with c being the relevant SU(2) spin ("fuzzyness"). Then it follows that c is quantized, $c \in \mathbb{Z}$. Actually, from the standpoint of the supergroup OSp(4|2), this $su(2)$ algebra is just a quantum version of the $su(2)$ generated by generators J^{ik} defined in (3.42).

The wave functions inherit this internal symmetry through a dependence on additional SU(2) spinor degrees of freedom. Let us consider the following realization for the operators Z^i and $\bar{Z}_i$

$$\bar{Z}_i = v_i^+\,, \qquad Z^i = \partial/\partial v_i^+ \tag{4.13}$$

where v_i^+ is a commuting complex SU(2) spinor. Then the constraint (4.2) on wave function $\Phi(x, v_i^+)$

$$D^0\Phi = \bar{Z}_i Z^i \Phi = v_i^+ \frac{\partial}{\partial v_i^+}\,\Phi = c\,\Phi \tag{4.14}$$

leads to the polynomial dependence of it on v_i^+:

$$\Phi(x, v_i^+) = \phi_{k_1\ldots k_c}(x) v^{+k_1} \ldots v^{+k_c}\,. \tag{4.15}$$

Thus, as opposed to the model of ref. [1], in our case the x-dependent wave function carries an irreducible spin $c/2$ representation of the group SU(2), being an SU(2) spinor of the rank c.

Using (4.7) and (4.12) we see that on physical states the quantum Hamiltonian is

$$\mathbf{H} = \frac{1}{4}\left(P^2 + \frac{g}{X^2}\right), \tag{4.16}$$

where

$$g = \frac{c}{2}\left(\frac{c}{2} + 1\right). \tag{4.17}$$

It is easy to show that the SU(1,1) Casimir operator takes the value $\frac{1}{4}\,g - \frac{3}{16}$ (for further details, see the next sections). Thus, like in [1], on the fields $\phi_{k_1\ldots k_c}(x)$ the unitary irreducible representations of the group SU(1,1) are realized, despite the fact that the wave function is now multi-component, with $(c+1)$ independent components. Requiring the wave function $\Phi(v^+)$ to be single-valued once again leads to the condition that $c \in \mathbb{Z}$. This quantization of parameter c could be important for the possible black hole interpretation of the considered variant of conformal mechanics.

Note that the new variables v_i^+ can be treated as a half of the target space harmonic-like variables v_i^+, v_i^- (though without the familiar constraint $v^{+i}v_i^- \sim const$). The harmonic interpretation could be made more literal using a different, mixed Dirac- Gupta-Bleuler quantization for z variables along the line of ref. [36].

4.2 Operator realization of OSp(4|2) superalgebra

Here we extend the bosonic-limit consideration to the whole OSp(4|2) mechanics.

Quantum operators of physical coordinates and momenta satisfy the quantum brackets, obtained in the standard way from (3.32) (by multiplying the latter by i):

$$[X, P] = i\,, \qquad [Z^i, \bar{Z}_j] = \delta^i_j\,, \qquad \{\Psi^i, \bar{\Psi}_j\} = -\frac{1}{2}\,\delta^i_j\,. \tag{4.18}$$

Quantum supertranslation and superconforml boost generators are uniquely defined by the classical expressions (3.30), (3.31). They appear to be linear in the odd operators:

$$\mathbf{Q}^i = P\Psi^i - i\frac{Z^{(i}\bar{Z}^{k)}\Psi_k}{X}\,, \qquad \bar{\mathbf{Q}}_i = P\bar{\Psi}_i + i\frac{Z_{(i}\bar{Z}_{k)}\bar{\Psi}^k}{X}\,, \tag{4.19}$$

$$\mathbf{S}^i = -2\,X\Psi^i + t\,\mathbf{Q}^i, \qquad \bar{\mathbf{S}}_i = -2\,X\bar{\Psi}_i + t\,\bar{\mathbf{Q}}_i\,. \tag{4.20}$$

Evaluating the anticommutators of the odd generators (4.19), (4.20), one determines uniquely the full set of quantum generators of superconformal algebra $D(2,1;-\frac{1}{2})$. We obtain

$$\mathbf{H} = \frac{1}{4}\,P^2 + \frac{(\bar{Z}_k Z^k)^2 + 2\bar{Z}_k Z^k}{16X^2} + \frac{Z^{(i}\bar{Z}^{k)}\Psi_{(i}\bar{\Psi}_{k)}}{X^2}\,, \tag{4.21}$$

$$\mathbf{K} = X^2 - t\,\frac{1}{2}\,\{X, P\} + t^2\,\mathbf{H}\,, \tag{4.22}$$

$$\mathbf{D} = -\frac{1}{4}\,\{X, P\} + t\,\mathbf{H}\,, \tag{4.23}$$

$$\mathbf{J}^{ik} = i\left[Z^{(i}\bar{Z}^{k)} + 2\Psi^{(i}\bar{\Psi}^{k)}\right], \tag{4.24}$$

$$\mathbf{I}^{1'1'} = -i\Psi_k\Psi^k\,, \qquad \mathbf{I}^{2'2'} = i\bar{\Psi}^k\bar{\Psi}_k\,, \qquad \mathbf{I}^{1'2'} = -\frac{i}{2}\,[\Psi_k, \bar{\Psi}^k]\,. \tag{4.25}$$

It can be directly checked that the generators (4.19)–(4.25) indeed form the $D(2,1;-\frac{1}{2})$ superalgebra which is obtained form the DB superalgebra (3.34)–(3.36) by changing altogether DB by (anti)commutators and multiplying the right-hand sides by i.

The second-order Casimir operator of $D(2,1;-\frac{1}{2})$ is given by the following expression [37]

$$\mathbf{C}_2 = \mathbf{T}^2 - \frac{1}{2}\,(\mathbf{J}^2 + \mathbf{I}^2) + \frac{i}{4}\,\mathbf{Q}^{ai'i}\mathbf{Q}_{ai'i} \tag{4.26}$$

(the quantum SU(1,1) generators $\mathbf{T}^{ab}$ are defined in terms of the generators (4.21)–(4.23) by the same formulas (3.38)). Using the relations

$$\mathbf{T}^2 \equiv \frac{1}{2}\mathbf{T}^{ab}\mathbf{T}_{ab} = \frac{1}{16}\left[(\bar{Z}_k Z^k)^2 + 2\bar{Z}_k Z^k\right] + Z^{(i}\bar{Z}^{k)}\Psi_{(i}\bar{\Psi}_{k)} - \frac{3}{16}\,, \tag{4.27}$$

$$\mathbf{J}^2 \equiv \frac{1}{2}\mathbf{J}^{ik}\mathbf{J}_{ik} = \frac{1}{4}\left[(\bar{Z}_k Z^k)^2 + 2\bar{Z}_k Z^k\right] - \frac{3}{2}\left(\Psi_i\Psi^i\,\bar{\Psi}^k\bar{\Psi}_k - \Psi_i\bar{\Psi}^i\right) - 2Z^{(i}\bar{Z}^{k)}\Psi_{(i}\bar{\Psi}_{k)}, \tag{4.28}$$

$$\mathbf{I}^2 \equiv \frac{1}{2}\mathbf{I}^{i'k'}\mathbf{I}_{i'k'} = \frac{1}{2}\{\bar{\mathbf{I}},\mathbf{I}\} - (\mathbf{I}_3)^2 = \frac{3}{2}\left(\Psi_i\Psi^i\,\bar{\Psi}^k\bar{\Psi}_k - \Psi_i\bar{\Psi}^i\right) + \frac{3}{4} \tag{4.29}$$

together with

$$\frac{i}{4}\,\mathbf{Q}^{ai'i}\mathbf{Q}_{ai'i} = \frac{i}{4}\,[\mathbf{Q}^i,\bar{\mathbf{S}}_i] + \frac{i}{4}\,[\bar{\mathbf{Q}}_i,\mathbf{S}^i] = -2Z^{(i}\bar{Z}^{k)}\Psi_{(i}\bar{\Psi}_{k)} + \frac{1}{2}\,, \tag{4.30}$$

we find that $\mathbf{C}_2$ takes the form

$$\mathbf{C}_2 = -\frac{1}{16}\left[(\bar{Z}_k Z^k)^2 + 2\bar{Z}_k Z^k + 1\right]. \tag{4.31}$$

Using (4.31), we can rewrite quantum Hamiltonian (4.21) in the following equivalent suggesting form:

$$\mathbf{H} = \frac{1}{4}\,P^2 - \frac{\mathbf{C}_2}{X^2} - \frac{1}{16X^2} + \frac{Z^{(i}\bar{Z}^{k)}\Psi_{(i}\bar{\Psi}_{k)}}{X^2}\,. \tag{4.32}$$

An important observation is that the following quantities belonging to the enveloping algebra of $osp(4|2)$ superalgebra

$$\mathbf{M} \equiv 4\mathbf{T}^2 - (\mathbf{J}^2 + \mathbf{I}^2) + \frac{3i}{4}\,\mathbf{Q}^{ai'i}\mathbf{Q}_{ai'i}\,, \tag{4.33}$$

$$\mathbf{M}^{ik,\,i'k'} \equiv \{\mathbf{J}^{ik},\mathbf{I}^{i'k'}\} + i\mathbf{Q}^{b(i'(i}\mathbf{Q}_b{}^{k')k)}\,, \tag{4.34}$$

$$\mathbf{M}^{ai'i} \equiv \frac{i}{2}\,\{\mathbf{T}^a_b,\mathbf{Q}^{bi'i}\} + \frac{i}{4}\,\{\mathbf{J}^i_j,\mathbf{Q}^{ai'j}\} + \frac{i}{4}\,\{\mathbf{I}^{i'}_{j'},\mathbf{Q}^{aj'i}\} \tag{4.35}$$

form a linear finite-dimensional representation of OSp(4|2):

$$[\mathbf{M},\mathbf{Q}^{ai'i}] = \mathbf{M}^{ai'i}\,, \qquad [\mathbf{M}^{ik,\,i'k'},\mathbf{Q}^{bj'j}] = -4\epsilon^{j(i}\epsilon^{j'(i'}\mathbf{M}^{ak')k)}\,,$$
$$\left[\mathbf{M}^{ai'i},\mathbf{Q}^{bk'k}\right] = -\frac{i}{2}\,\epsilon^{ab}\epsilon^{i'k'}\epsilon^{ik}\mathbf{M} - \frac{i}{2}\,\epsilon^{ab}\mathbf{M}^{ik,\,i'k'}\,.$$

For the particular representation of generators given by eqs. (4.27)–(4.29) all quantities (4.33)–(4.35) identically vanish:

$$\mathbf{M} = 0\,, \qquad \mathbf{M}^{ik,\,i'k'} = 0\,, \qquad \mathbf{M}^{ai'i} = 0\,. \tag{4.36}$$

As a consequence of these identities, there arises the relation

$$\mathbf{T}^2 + \frac{1}{2}\,(\mathbf{J}^2 + \mathbf{I}^2) = -3\mathbf{C}_2\,. \tag{4.37}$$

Thus, for an irreducible representation of $D(2,1;-\frac{1}{2})$ with a fixed $\mathbf{C}_2$ (see (4.47) below) the values of the Casimir operators $\mathbf{T}^2$, $\mathbf{J}^2$, $\mathbf{I}^2$ of three bosonic subgroups $sl(2,\mathbb{R})$, $su(2)$, $su'(2)$ prove to be related as in (4.37).

4.3 Quantum spectrum

The Hamiltonian (4.21) and the $sl(2,\mathbb{R})$ Casimir operator (4.27) can be represented as

$$\mathbf{H} = \frac{1}{4}\left(P^2 + \frac{\hat{g}}{X^2}\right), \tag{4.38}$$

$$\mathbf{T}^2 = \frac{1}{4}\,\hat{g} - \frac{3}{16}\,, \tag{4.39}$$

where

$$\hat{g} \equiv \frac{1}{2}\,\bar{Z}_k Z^k \left(\frac{1}{2}\,\bar{Z}_k Z^k + 1\right) + 4Z^{(i}\bar{Z}^{k)}\Psi_{(i}\bar{\Psi}_{k)}\,. \tag{4.40}$$

The operators (4.38) and (4.39) formally look like those given in the $\mathrm{SU}(1,1)$ model of [1]. However, there is an essential difference. Whereas the quantity $\hat{g}$ is a constant in the $\mathrm{SU}(1,1)$ model, in our case $\hat{g}$ is an operator taking fixed, but different, constant values on different components of the full wave function.

To find the quantum spectrum of (4.38) and (4.39), we make use of the realization (4.13) for the bosonic operators Z^k and $\bar{Z}_k$, as well as the following realization of the odd operators Ψ^i, $\bar{\Psi}_i$

$$\Psi^i = \psi^i, \qquad \bar{\Psi}_i = -\frac{1}{2}\,\partial/\partial\psi^i\,, \tag{4.41}$$

where ψ^i are complex Grassmann variables. Then, the state vector (wave function) is defined as

$$\Phi = A_1 + \psi^i B_i + \psi^i\psi_i A_2\,. \tag{4.42}$$

The full wave function is subjected to the same constraints (3.44) as in the bosonic limit (we use the normal ordering for even SU(2)–spinor operators, with all operators Z^i standing on the right)

$$D^0\Phi = \bar{Z}_i Z^i \Phi = v_i^+ \frac{\partial}{\partial v_i^+}\,\Phi = c\,\Phi. \tag{4.43}$$

Like in the bosonic limit, requiring the wave function $\Phi(v^+)$ to be single-valued gives rise to the condition that the constant c must be integer, $c \in \mathbb{Z}$. We take c to be positive in order to have a correspondence with the bosonic limit where c becomes SU(2) spin. Then (4.43) implies that the wave function $\Phi(v^+)$ is a homogeneous polynomial in v_i^+ of the degree c:

$$\Phi = A_1^{(c)} + \psi^i B_i^{(c)} + \psi^i\psi_i A_2^{(c)}\,, \tag{4.44}$$

$$A_{i'}^{(c)} = A_{i',k_1\ldots k_c} v^{+k_1}\ldots v^{+k_c}\,, \tag{4.45}$$

$$B_i^{(c)} = B_i^{\prime(c)} + B_i^{\prime\prime(c)} = v_i^+ B'_{k_1\ldots k_{c-1}} v^{+k_1}\ldots v^{+k_{c-1}} + B''_{(ik_1\ldots k_c)} v^{+k_1}\ldots v^{+k_c}\,. \tag{4.46}$$

In (4.46) we extracted SU(2) irreducible parts $B'_{(k_1\ldots k_{c-1})}$ and $B''_{(ik_1\ldots k_c)}$ of the component wave functions, with the SU(2) spins $(c-1)/2$ and $(c+1)/2$, respectively.

	r_0	j	i
$A^{(c)}_{k'}(x,v^+)$	$\frac{c+3}{4}$	$\frac{c}{2}$	$\frac{1}{2}$
$B'^{(c)}_{k}(x,v^+)$	$\frac{c+3}{4}+\frac{1}{2}$	$\frac{c}{2}-\frac{1}{2}$	0
$B''^{(c)}_{k}(x,v^+)$	$\frac{c+3}{4}-\frac{1}{2}$	$\frac{c}{2}+\frac{1}{2}$	0

Table 1. The $\mathrm{SU}(1,1)$, $\mathrm{SU}(2)$ and $\mathrm{SU}'(2)$ quantum numbers.

On the physical states (4.43), (4.44) Casimir operator (4.31) takes the value

$$\mathbf{C}_2 = -(c+1)^2/16\,. \tag{4.47}$$

On the same states, the Casimir operators (4.27)–(4.29) of the bosonic subgroups $\mathrm{SU}(1,1)$, $\mathrm{SU}(2)$ and $\mathrm{SU}'(2)$ take the following values

$$\mathbf{T}^2 = r_0(r_0-1)\,, \qquad \mathbf{J}^2 = j(j+1)\,, \qquad \mathbf{I}^2 = i(i+1)$$

For different component wave functions, the quantum numbers r_0, j and i take the values listed in table 1.

The fields B'_i and B''_i form doublets of $\mathrm{SU}(2)$ generated by $\mathbf{J}^{ik}$, whereas the component fields $A_{i'} = (A_1, A_2)$ form a doublet of $\mathrm{SU}'(2)$ generated by $\mathbf{I}^{i'k'}$. If the super-wave function (4.42) is bosonic (fermionic), the fields $A_{i'}$ describe bosons (fermions), whereas the fields B'_i, B''_i present fermions (bosons). It is easy to check that the constraint (4.37) is satisfied in all cases.

Each of the component wave functions $A_{i'}$, B'_i, B''_i carries an infinite-dimensional unitary representation of the discrete series of the universal covering group of the SU(1,1) one-dimensional conformal group. Such representations are characterized by positive numbers r_0 [38, 39] (for the unitary representations of SU(1,1) the constant $r_0>0$ must be (half)integer). Basis functions of these representations are eigenvectors of the compact SU(1,1) generator

$$\mathbf{R} = \frac{1}{2}\left(a^{-1}\mathbf{K} + a\mathbf{H}\right),$$

where a is a constant of the length dimension. These eigenvalues are $r = r_0 + n$, $n \in \mathbb{N}$ [1, 38, 39].

Using the expressions (4.21), (4.32), (4.47) we can write he Hamiltonian in the form, common for all component wave functions,

$$\mathbf{H} = \frac{1}{4}\left(P^2 + \frac{l(l+1)}{X^2}\right) \tag{4.48}$$

where constant l takes the values given in table 2.

Let us focus on some peculiar properties of the OSp(4|2) quantum mechanics constructed.

As opposed to the $\mathrm{SU}(1,1|2)$ superconformal mechanics [12–14], the construction presented here essentially uses the variables z_i (or v^+_i) parametrizing the two-sphere S^2, in addition to the standard (dilatonic) coordinate x.

JHEP08(2009)081

	l
$A^{(c)}_{k'}(x, v^+)$	$\frac{c}{2}$
$B'^{(c)}_{k}(x, v^+)$	$\frac{c}{2}+1$
$B''^{(c)}_{k}(x, v^+)$	$\frac{c}{2}-1$

Table 2. Values of the constant l.

Presence of additional "spin" S^2 variables in our construction leads to a richer quantum spectrum: the relevant wave functions involve representations of the two independent SU(2) groups, in contrast to the $\mathrm{SU}(1,1|2)$ models where only one SU(2) realized on fermionic variables matters.

Also in a contradistinction to the previously considered models, there naturally appears a quantization of the conformal coupling constant which is expressed as a SU(2) Casimir operator, with both integer and half-integer eigenvalues. This happens already in the bosonic sector of the model, and is ensured by the S^2 variables.

5 Summary and outlook

We have investigated $\mathcal{N}$=4 superconformal mechanics with $\mathrm{OSp}(4|2)$ symmetry. This model is the one-particle case (or the center-of-mass sector) of a $\mathcal{N}$=4 superconformal Calogero model recently proposed in [28]. After eliminating the auxiliary and gauge degrees of freedom, we obtained the $\mathrm{OSp}(4|2)$ generators both on the classical and on the quantum level.

The physical sector of the model is described by one "radial" coordinate x, four Grassmann-odd fermionic coordinates ψ_i and $\bar\psi_i$ as well as a Grassmann-even SU(2) doublet z_i which parameterizes S^2. The latter lack a standard kinetic term and appear only in a Wess-Zumino term, i.e. to first order in time derivatives. These SU(2) spinor variables lead to an unusual but rather nice property: the odd $\mathrm{OSp}(4|2)$ generators are *linear* in ψ_i or $\bar\psi_i$, as opposed to $\mathrm{SU}(1,1|2)$ superconformal mechanics [12–14][3] where such generators require also terms cubic in the fermions. Note that $\mathcal{N}$>4 supersymmetric mechanics with linear supercharges is trivial as was indicated in [40].

We observed an interesting feature which might be called a "double harmonic extension". At the classical level, the worldline parameter t is extended by harmonic variables $u^{\pm}_i$. The above-mentioned SU(2) spinor variables z^i can be interpreted as a kind of harmonic target variables, in line with [36]. The corresponding quantum operators Z^i serve as coordinates of a fuzzy sphere.

We performed an analysis of the quantum spectrum. Its form relates to a subspace in the enveloping algebra of $osp(4|2)$ which is closed under the $osp(4|2)$ action. The composite generators from this set turn out to vanish for the specific realization of the $osp(4|2)$ superalgebra pertinent to our model.

[3]The general supergroup $D(2,1;\alpha)$ was apparently implicit in [14].

Finally, let us discuss the links of our model to the black-hole and AdS/CFT story. The Hamiltonian (4.48) resembles the Hamiltonian for the radial motion of a massive charged particle near the horizon of an extremal RN black hole [4] in the supersymmetry-preserving (or BPS) limit, when the mass and electric charge of the superparticle are equal. In our model, the quantized "angular momentum",[4] whose square is the strength of the conformal potential, is given by the bosonic SU(2)-spinors and is present already in the bosonic sector. It receives corrections from the SU(2)-spinor fermions for other components of the wave superfunction.

Despite this formal resemblance, the superconformal symmetries differ: it is SU(1,1|2) for the near-horizon limit of the RN black-hole solution of $\mathcal{N}=2, d=4$ supergravity, while our model is OSp(4|2) invariant. Thus, one may ask to which sort of superparticle our superconformal mechanics does correspond. This can be explored by changing variables to the so-called AdS basis [9, 15, 20, 41], in which the $d=1$ conformal group $\mathrm{SO}(1,2)$ is realized by relativistic particle motions on $\mathrm{AdS}_2 \simeq \mathrm{SO}(1,2)/\mathrm{SO}(1,1)$. In addition, the Wess-Zumino term in the action (3.21) describes the coupling of a charged particle on S^2 to a Dirac monopole in its center. The strength of the Wess-Zumino term is given by the product of the electric and magnetic charges of the particle and monopole, respectively. The potential for this magnetic flux is naturally present in the general form of the RN solution (along with an electric potential). However, in our case the S^2 variables are not propagating in either the conformal or the AdS bases. Therefore, the hypothetical superparticle associated with our superconformal model moves only on the AdS_2 space and not on the $\mathrm{AdS}_2 \times S^2$ appearing for SU(1,1|2) mechanics. On the other hand, the presence of the Wess-Zumino term suggests that our superparticle still couples to the magnetic charge. It would be interesting to inquire whether a background with such properties can arise in a black-hole type supergravity solution in higher dimensions. Since the off-shell content of our model contains four bosonic degrees of freedom plus the worldline time for a fifth variable, we conjecture that the appropriate supergravity should live in five spacetime dimensions.

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Note added. After finishing this paper, related work [42] overlapping with ours appeared in the arXiv.

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